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Stabilization of Networked Control System with Packet Dropouts

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Abstract—Time delays in exchange of information and packet dropoutsare very common in NCS (Feedback control systems with communications networks for data exchange). Owing to the occurrence of such time delays and packet dropouts, the NCS systems are uncertain and time-varying. Also these time dealys and packet dropouts may result in degartion of the NCS performance and instability.This paper presents a new design of a state-feedback controller usingLyapunov stability criterion in terms of LMI conditions for the time varying uncertain NCS. Further, the LMI condition is used to guarantee the stability of the NCS. Results are included to demonstrate this presented approach to stabilization of NCS.

Index Terms—Communication delay, Lyapunov stability, NCS, nominal value, packet dropouts, state feedback controller,uncertainty and Zero Order Hold (ZOH).

1 Introduction

THE point-to-point architecture is the traditional communication architecture for control systems [1], i.e. sensors and actuators are connected to controllers via wires. Due to advances in network communications the traditional point-to-point architecture is no longer used. Instead, to meet new requirements, such as modularity, integrated diagnostics, quick and easy maintenance the recent industrial control schemes explit the communication networks in the feedback control systems.Further, the common-bus network architectures can improve the efficiency, flexibility and reliability of integrated applications, and reduce installation, reconfiguration, maintenance time and costs [3,7]. In recent years, therefore, it gives rise to the so-called Network-Based Control Systems (NCSs) [2, 3] as shown in Fig.1.The challenge lies in designing NCSs are in incorporating the issues like variable time-delays and packet dropouts introduced by use of real-time communication network in the system model and thereby designing the controller. Moreover, for controlling a continuous-time plant over a digital communication network (shown in Fig. 1.) offers more complexity in analysis and design. The plant is a continuous one whereas the feedback control is through digital network. The overall system is remodelled in discrete-domain. The ZOH reconstructs analog singal from the discrete signals. Networks introduced in feedback control loop raises two fundamental issues known as data packet dropouts and delay in receiving of measurement data and control data across the communication channel [8]. These issues may be approached individually or in a combination. The objective of the work described in this paper is to deal with the packet dropouts and the uncertainties arising out in the NCS due to insertion of network.Packet dropouts in NCS are always a focused problem for researchers. Due to time delay and packet dropouts, there may be unavailability of measurement data at controller end or control data at actuator end. Thus, the uncertainty in the system results in instability [3].

Dropouts in a system can happen in any state, may be dropouts in past state or in future state of the system. This paper describes the combined effect of the dropouts, i.e.considering the system with all possible types of dropouts as a whole and stabilizing the system. This gives a robustness to the system that can tolerate any possible dropouts occurring randomly.

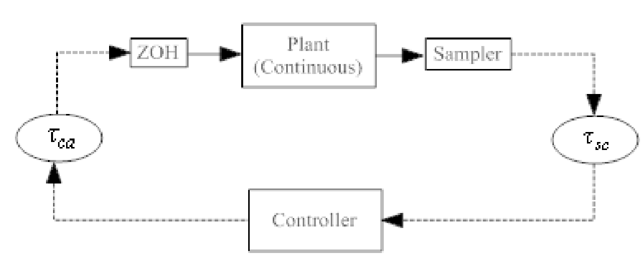

Fig.1 Structure of a Networked Control System

Section 2presents the modeling of NCS with uncertainty and derives a sufficient stabilization condition using Lyapunov stability criterion. Section 3 presents results and discussions. Finally, section 4 concludes the paper.

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2 System Model

As in a NCS the data is being sent and received in digital form, it is necessary to go for discrete domain for easy analysis. The plant dynamics in continuous form is given by

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$$\dot{x}(t) = Ax(t) + Bu(t) \quad \text{and } u(t) = Kx(t) \qquad (1)$$

where $x(t) \in R^n$ is the state vector and $u(t) \in R^m$ the plant input; A, B are known matrices and K is feedback gain to be designed. Continuous to discrete conversion methods [5] includes finding the state transition matrix (solution of e^{AT}), where A = system matrix, T = sampling interval. Computation of $F = e^{AT}$ can be evaluated using the formula

$$e^{-AT} = 1 + \frac{AT}{1!} + \frac{A^2T^2}{2!} + \frac{A^3T^3}{2!} \ldots \qquad (2)$$

and computation of G will be as,

$$G = \int_0^T e^{-As}Bds. = (e^{-AT} - I)A^{-1}B \qquad (3)$$

A discrete system with state feedback control can be described as follows.

$$x(k+1) = Fx(k) + Gu(k) \quad \text{and } u(t) = Kx(k) \qquad (4)$$

where $x(k) \in R^n$, $u(k) \in R^m$ are the state and input vectorin discrete domain; F, G are known matrices.When a system is used as a part of NCS there may uncertainty lies due to packet loss across the network. Due to this, the system matrix F and G mixed with uncertainties denoted as ΔF and ΔG .Augumenting the uncertainty terms in state vector(4) can be written as

$$x(k+1) = (F + \Delta F)x(k) + (G + \Delta G)u(k) \qquad (5)$$

Let ΔF denotes the uncertainties associated with F . For each sampling interval (T) there is a F matrix assosiciated with a system. For one sample intervalT there exits a system matrix F_1, similarly for n number of sample intervals we have F_n.Transmission interval is defined as the total time required from generation of a packet at sensor side to receive that packet at plant side to generate a new signal via controller and actuator. The relation between sampling instance, transmission interval, and packet is shown in Fig.2.

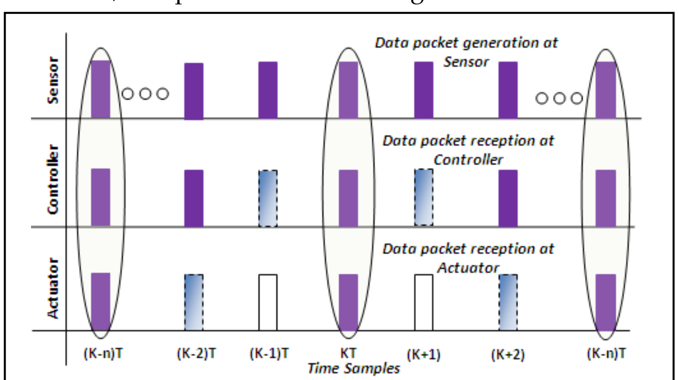

Fig.2. Illustration of packet loss, transmission interval in NCS

Fig.2 illustrates the transmission interval with respect to packet loss. The dark bar at sensor end shows the data packet generation, the dark bar at controller and actuator shows the reception of data packets. The dashed bars show the packet is being lost in transmission. The white bars shows packets are not received. No of sampling instances required for a complete transmission interval denotes the no of packet losses. e.g. in Fig.2, at $(K-n)T, KT$ and $(K+n)T$ there is no packet loss, as the packet generated at sensor is received at actuator via controller to generate another measurement data. The transmission interval is from $(K-n)T$ to KT and from KT to $(K+n)T$. These two transmission intervals also show the amount of packet loss as shown in Fig.2. The next objective is to find the amount of uncertainty added in system matrices due to the packet losses in the respective transmission intervals.

Computation of F and ΔF

For each sampling interval, T ,thereis an associated system matrix (F) . According to (2)it can be written as-

$$F_n = e^{nAT} = 1 + \frac{A(nT)}{1!} + \frac{A^2\{(n)T\}^2}{2!} + \ldots \frac{A^N(nT)^N}{N!} \qquad (6)$$

N = maximum number of iteration at which the series tends to zero.

Nominal F value:

Let the amount of packet loss considered at m^{th} sampling instance is $m-1$, then F_m is known as the nominal F value. Other values of F above and below of m are known as uncertainty values. For $\S = 0,1,-1$ we can have the generalized formula.

$$F_{m+\S} = e^{(m+\S)AT} = 1 + \frac{A\{(m+\S)T\}}{1!} + \frac{A^2\{(m+\S)T\}^2}{2!} + \ldots \frac{A^N\{(m+\S)T\}^N}{N!} \qquad (7)$$

The summation of (7) for different values of § will give the F matrix with uncertainties, which can be expressed as

$$F + \Delta F = 1 + \frac{A\{(m+\S)T\}}{1!} + \frac{A^2\{(m+\S)T\}^2}{2!} + \ldots \frac{A^N\{(m+\S)T\}^N}{N!} \qquad (8)$$

From Eq.(8) it is seen that when § equals to zero it is the nominal and for other values it is uncertainty. F_{m+1} is the maximum, F_{m-1} is the minimum and F_m is the nominal value of F . So (8) can be written as

$$F + \Delta F = 1 + \frac{A(mT)}{1!} + \frac{A^2\{(m)T\}^2}{2!} + \ldots + \frac{A\{(\S)T\}}{1!} + \frac{A^2\{(2m\S)T + \S^2T^2}{2!} + \ldots \qquad (9)$$

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where $(\S)T = \S_{1f}, (2m\S)T + \S^2 T^2 = \S_{2f}$, are uncertainties and $\overline{\S_{1f}}, \overline{\S_{2f}}$ are the maximum values of uncertainty that is associated with the system. The generalized value of $\S_Q$ can be written as-

$$\S_{Qf} = \{(m+\S)T\}^Q - (mT)^Q \qquad (10)$$

where $Q = 1,2,3\ldots\ldots N$ and the maximum values of $\S_Q$ denoted $\overline{\S_Q}$. From (8),one can find the value of ΔF as-

$$\Delta F = F_{MAXIMUM} - F_{NOMINAL}$$

Structural exploration of ΔF

The uncertainty matrix ΔF can be written as-

$$\Delta F = D_f f_f E_f$$

$$= \left[\overline{\S_{1f}}A \; \overline{\S_{2f}}\frac{A^2}{2!} \; \overline{\S_{3f}}\frac{A^3}{3!} \ldots \right] \left[\begin{bmatrix} \frac{\S_{1f} I}{\overline{\S_{1f}}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\S_{Nf} I}{\overline{\S_{Nf}}} \end{bmatrix} \right] I \qquad (11)$$

For a $r \times r$ matrix A the size of D_f is $r \times N$, f_f is a diagonal matrix of $N \times N$ and E_f is an identity matrix of $N \times r$, and $\bar{\S}$ is the maximum value of uncertainty that is associated with the system and I is an identity matrix.

Computation of G and ΔG

From (3),one can write $G = (e^{-AT} - I)A^{-1}B$ so as the case of F we can write-

$$G_n = (1 + \frac{A\ -nT}{1!} + \frac{A^2\ -nT\ ^2}{2!} + \ldots - I)A^{-1}B \qquad (12)$$

Nominal G value

Let the amount of packet loss considered at the m^{th} sampling instance is $m-1$ then G_m is known as the nominal G value. Other values of G above and below of m is known as uncertainty values.

$$G_{m+\S} = e^{(m+\S)AT} = \left(1 + \frac{A\ \ m+\S\ T}{1!} + \frac{A^2\{(m+\S)T\}^2}{2!} + \ldots \frac{A^N\ \ m+\S)T\ ^N}{N!} - \mathrm{I}\right)\mathrm{A}^{-1}\mathrm{B} \qquad (13)$$

The summation of (13)for all values of § will give G and ΔG is calculated as follows-

$$\Delta G = G_{MAXIMUM} - G_{NOMINAL}$$

The uncertainties $\S_{Qg}$ and maximum uncertainties $\overline{\S_{Qg}}$ associated with G matrix can be found in the similar way asin(10).

Structural exploration of ΔG

The uncertainty matrix ΔG can be written as-

$$\Delta G = D_g f_g E_g$$

$$= \left[\overline{\S_{1g}}A \; \overline{\S_{2g}}\frac{A^2}{2!} \; \overline{\S_{3g}}\frac{A^3}{3!} \ldots \right] \left[\begin{bmatrix} \frac{\S_{1g} \mathrm{I}}{\overline{\S_{1g}}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\S_{Ng} \mathrm{I}}{\overline{\S_{Ng}}} \end{bmatrix} \right] I \qquad (14)$$

For a $r \times r$ matrix A and $r \times d$ matrix B the size of D_g is $r \times N$, f_f is a diagonal matrix of $N \times N$ and E_f is an identity matrix of $N \times d$,. Where $\S_{1g}$ and $\overline{\S_{Ng}}$ are the uncertainty and maximum value of uncertainty respectively that is associated with the system. I is the identity matrix.

Considering a Lyapunovcandidate function as

$$V(k) = x_k^T P x_k \qquad (15)$$

where P is positive definite matrix, in accordance to Lyapunov stability criteria [4, 6] the above equation (15) tends to asymptotic stable if it satisfy-

$$\Delta V = \left[F + \Delta F^{\ T} + K^T\ G + \Delta G^{\ T} \right] P \left[F + \Delta F\ + G + \Delta G\ K - P \right] < 0 \qquad (16)$$

the(16)can be written as-

$$\begin{bmatrix} -P & \left[F + \Delta F^{\ T} + K^T\ G + \Delta G^{\ T} \right] P \\ P\left[F + \Delta F\ + G + \Delta G\ K \right] & -P \end{bmatrix} < 0$$

$$\begin{bmatrix} -P & F + GK^{\ T} P \\ P\ F + GK & -P \end{bmatrix} + \begin{bmatrix} \varepsilon_f^{-1} E_f^{\ T} E_f + \varepsilon_g^{-1} K E_g^{\ T} E_g K & 0 \\ 0 & \varepsilon_f P D_f D_f^T P + \varepsilon_g P D_g D_g^T P \end{bmatrix} \qquad (17)$$

Multiply both sides by diagonal matrix (P^{-1}) in both sides of above equation (18),one obtaines

$$\begin{bmatrix} -P^{-1} + \varepsilon_f^{-1} P^{-1} E_f^{\ T} E_f P^{-1} + \varepsilon_g^{-1} P^{-1} K^T E_g^{\ T} E_g K P^{-1} & P^{-1}(F+GK)^T \\ (F+GK)P^{-1} & -P^{-1} + \varepsilon_f D_f D_f^T + \varepsilon_g D_g D_g^T \end{bmatrix} \qquad (18)$$

Assuming $P^{-1} = X$ and $K^T = Y^T$ and using Schur compilation, the LMI is obtained as

$$\begin{bmatrix} -X & XF^T + Y^T G^T & XE_f^{\ T} & Y^T E_g^{\ T} \\ * & -X + \varepsilon_f D_f D_f^T + \varepsilon_g D_g D_g^T & 0 & 0 \\ * & * & -\varepsilon_f I & 0 \\ * & * & * & -\varepsilon_g I \end{bmatrix} < 0 \qquad (19)$$

where * is transpose of a corresponding value.

3 RESULTSAND DISCUSSION

The transfer function $G(s)$ of the DC servo motor is $\frac{53.2718}{s^3 + 9.481s^2 + 36.18s + 0.8211}$ and in the state space parameters (A, B, C, D) can be obtained using MATLAB, where A=[-

9.4810,-36.1800,-0.8211;1,0,0;0,1,0], B=[1;0;0],C=[0,0,53.2718] and D=0;

Considering the maximum packet loss to be 7 and finding the system matrices F and G as per (6)and(12) for each instance of packet loss, with $\S = 0,1,-1$, we obtain D_f, D_g, E_f and E_g using (11)and (14). D_f is matrix of 3×90 and E_f is identity matrix of 90×3 and D_g is matrix of 3×30 and E_g is identity matrix of 30×3. After finding the all above values of we have to solve for LMI of (19)for unknowns like X, Y, ε_f and ε_g . After solving and as previously assumed that, the system has maximum packet loss 7, the maximum uncertainty values associated with F and G matrix are found to be as $\varepsilon_f = 23.3080, \varepsilon_g = 23.3144$ respectively.

Fig.3 shows the gradual increase of system uncertainties with packet loss. Fig.4 shows that the system statesapproach to the equliburium point irrespective to packet loss.

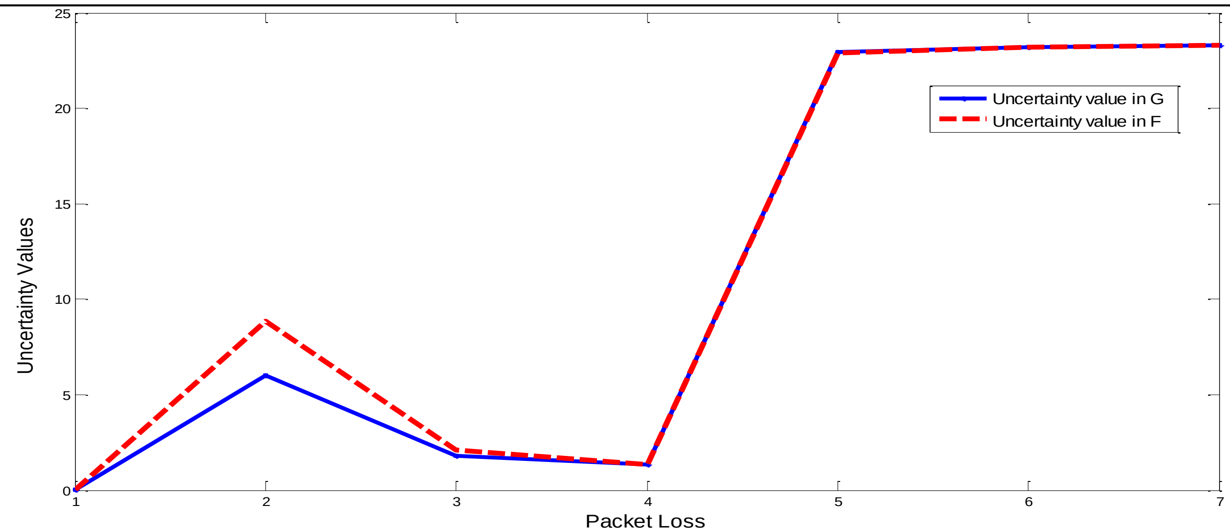

Fig. 3. Efffect of Packet droputs on Uncertainty

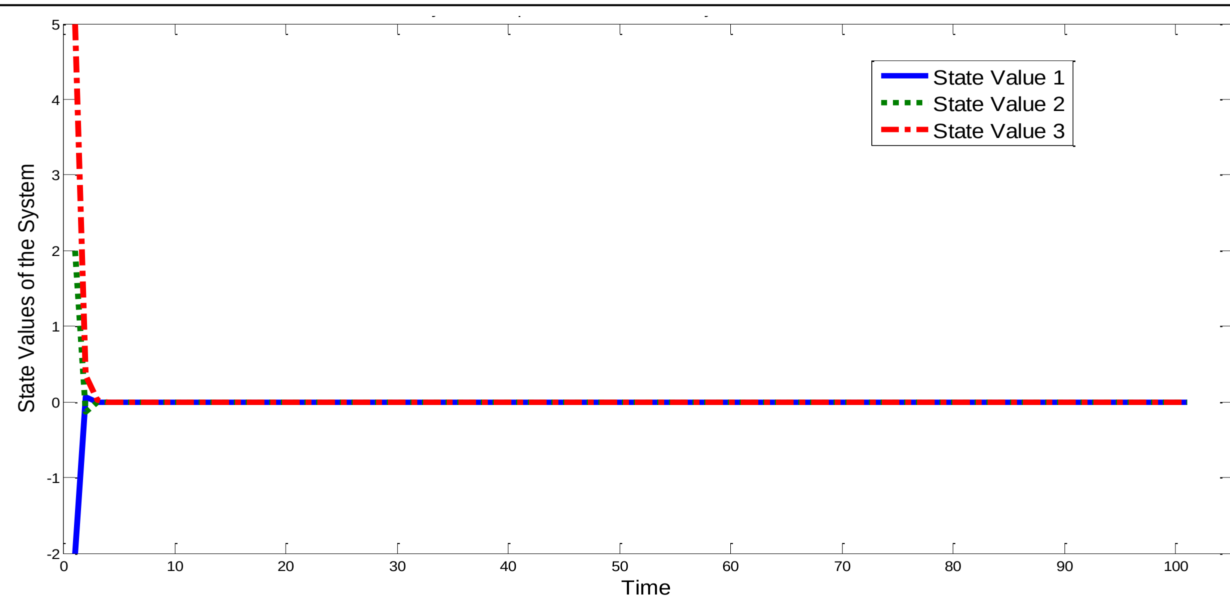

Fig.4. System State Response .

4 Conclusions

Uncertainty in a NCS due to packet loss is explained in this paper considering different transmission intervals. The uncertainty analysis of the studied NCS is carried out to deal with its stabilization. A sufficient condition for stabilization is derived using LMI. A state feedback controller is also implemented to ensure the stabilization of the NCS.

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